Volatility-constrained correlationを用いた金融市場間の影響伝播の解析

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Introduction

- Recent financial crises have shown the importance of determining the directionality of the influence between financial assets in order to identify the origin of market instabilities.
- Here, we analyze the correlation between Japan's Nikkei stock average index (Nikkei 225) and other financial markets by introducing a volatility-constrained correlation metric.
- The asymmetric feature of the metric reveals which asset is more influential than the other. As a result, this method allows us to unveil the directionality of the correlation effect, which could not be observed from the standard correlation analysis.

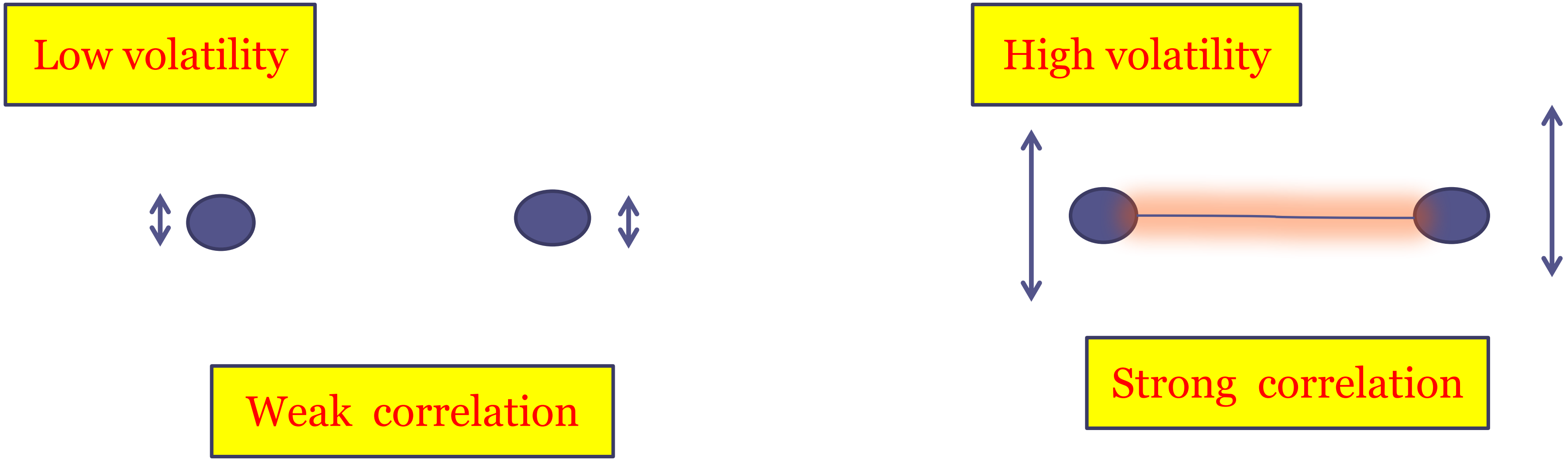
Background(1)

- Pearson's correlation coefficient is very standard for analysing a correlation between two asset returns.

$$C(R_1(t), R_2(t)) = \frac{1}{(t_f - t_i)} \sum_{t_i \leq t < t_f} \frac{(R_1(t) - E(R_1(t)))}{\sigma(R_1(t))} \frac{(R_2(t) - E(R_2(t)))}{\sigma(R_2(t))}.$$

Background(2)

In several works, it is reported that high volatile markets are directly related to strong correlations between them. (for example, T. Preis, et al, Scientific Reports 2 (2012) 752.)



Question

- The concept of standard correlation coefficient Cor(A, B) between A and B is symmetric by exchanging the two variables A and B.
- Therefore, most of the correlation research have not captured the directionality of the influence.
- Once we know that two asset returns is correlated with each other, is it possible to detect the directionality of the correlation effect?

yes

Method

I. Compute the standard correlation (Pearson's correlation coefficient) between the log returns of two assets (R1(t) and R2(t)) and check that they have a correlation.

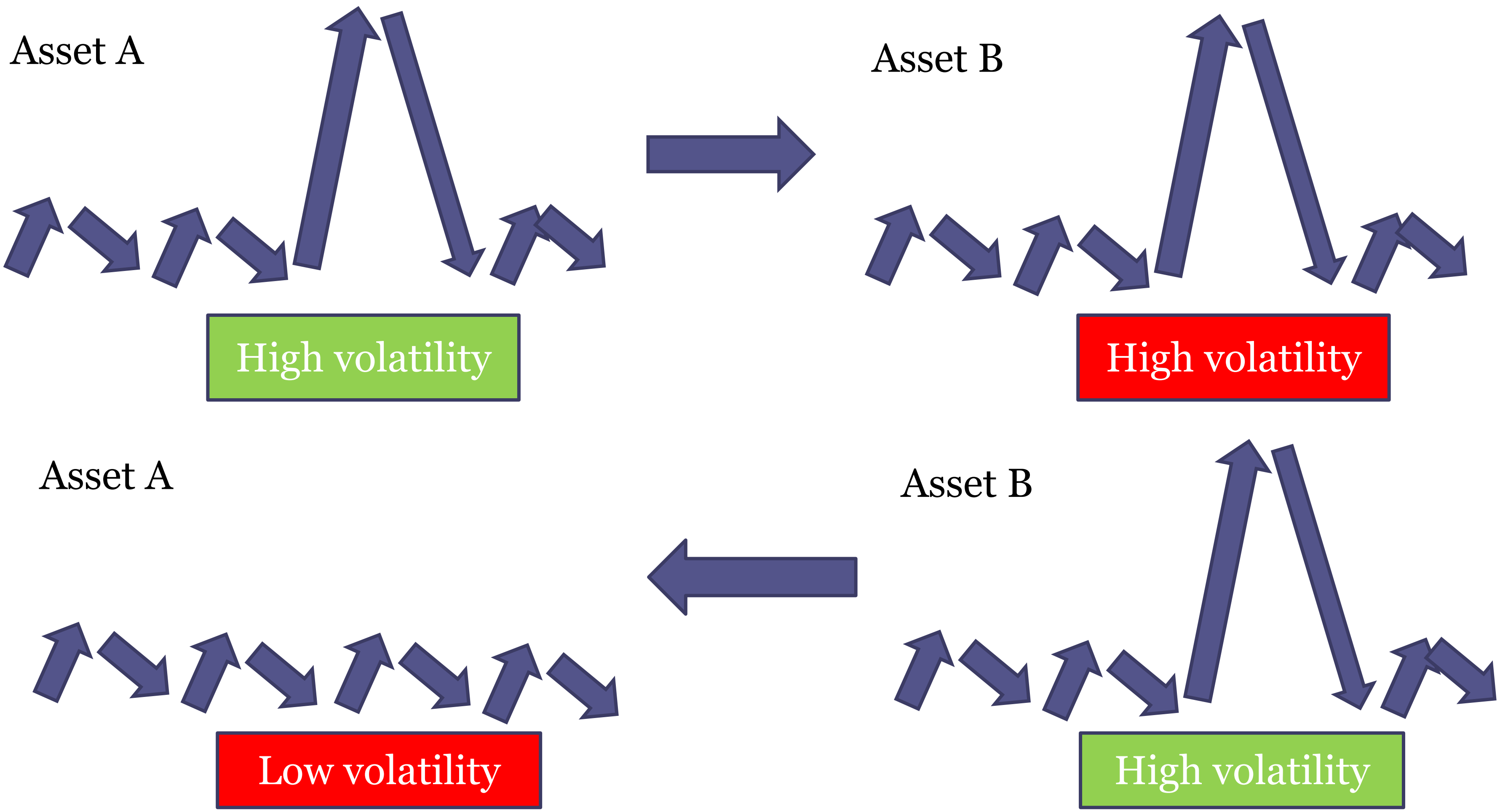
II. Filter the time series data, by selecting data where the returns R1(t) are more than its standard deviation (1 σ) . After that, compute the constrained correlation using this filtered time series data. (Base asset is R1(t))

III. Repeat II, but for R2(t). (Base asset is R2(t))

IV. Compare the constrained correlation of base asset R1(t) with that of base asset R2(t) . (Compare II and III)

Main Idea

Asset A is more influential to Asset B



Metrics

$$E(R(t),\Omega) = \frac{1}{\#\Omega}\sum_{t\in\Omega} R(t),$$

Constrained Expection

$$\sigma(R(t),\Omega) = \sqrt{\frac{1}{\#\Omega}\sum_{t\in\Omega}(R(t) - E(R(t),\Omega))^2},$$

Constrained SD

$$C(R_1(t),R_2(t),\Omega) = \frac{1}{\#\Omega}\sum_{t\in\Omega}\frac{(R_1(t) - E(R_1(t,\Omega)))}{\sigma(R_1(t),\Omega)}\frac{(R_2(t) - E(R_2(t,\Omega)))}{\sigma(R_2(t),\Omega)},$$

Constrained correlation

where

$$\Omega_{[t_1,t_2;\alpha,\beta]} = \{t \in [t_i, t_f] \mid t_1 \le t < t_2 \text{ and } \alpha\cdot\sigma(R_1(t)) \le |R_1(t)| < \beta\cdot\sigma(R_1(t))\},$$

Volatility constrained correlation

Let us consider the correlation of log returns of two assets R1(t) and R2(t) . Here we define new metric for determine the directionality of two asset returns.

$$F[\alpha, \beta](s) = C(R_1(t), R_2(t), \Omega_{[s,s+\Delta s;\alpha,\beta]}),$$

It represents a constrained correlation between R1(t) and R2(t) for each period in which the data set (R1(t),R2(t)) is constrained such that |R1(t)| is limited to a specific range (i.e. $\alpha \cdot \sigma(R1(t)) \leq |R1(t)| < \beta \cdot \sigma(R1(t))$).

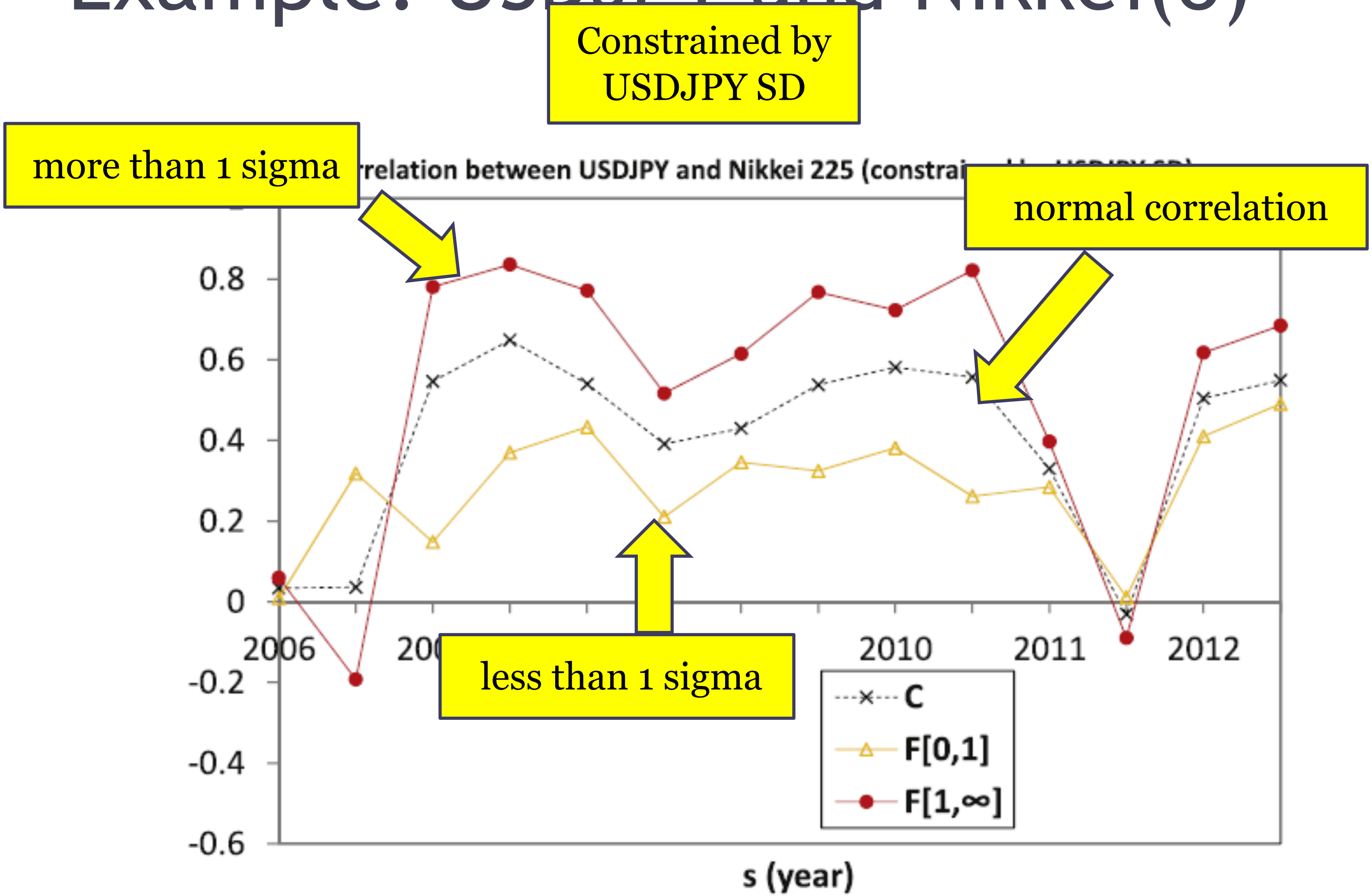
Data sets

We use the daily historical data of

- Japan's Nikkei 225 stock average index (Nikkei 225) ,
- Dow Jones Industrial Average (DJIA)
- the foreign exchange rate from the United States Dollar to the Japanese Yen (USDJPY),

for the period from 2006 to 2012.

Example: USDJPY and Nikkei(o)



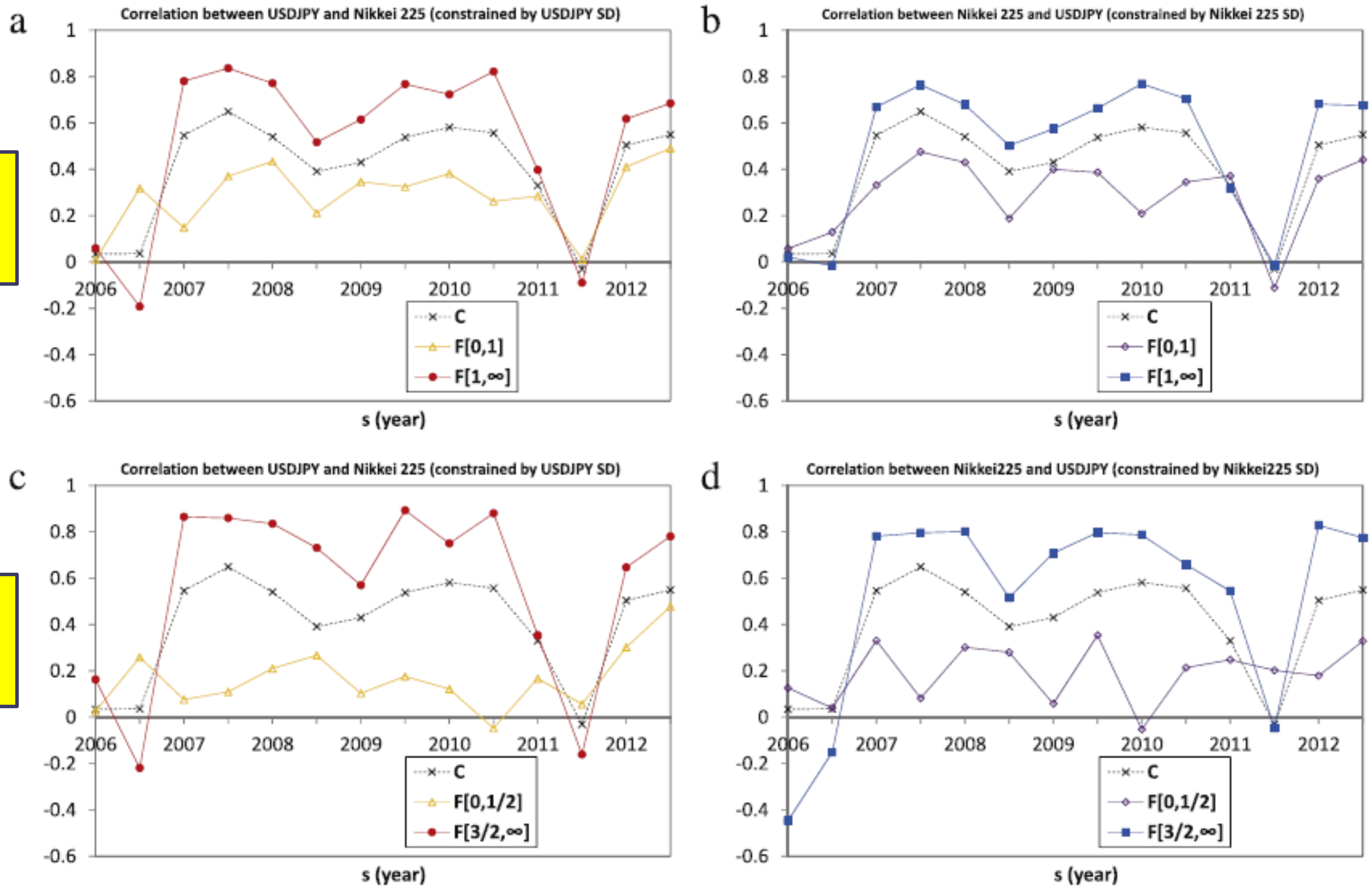
Empirical result: USDJPY and Nikkei(o)

Constrained by USDJPY SD

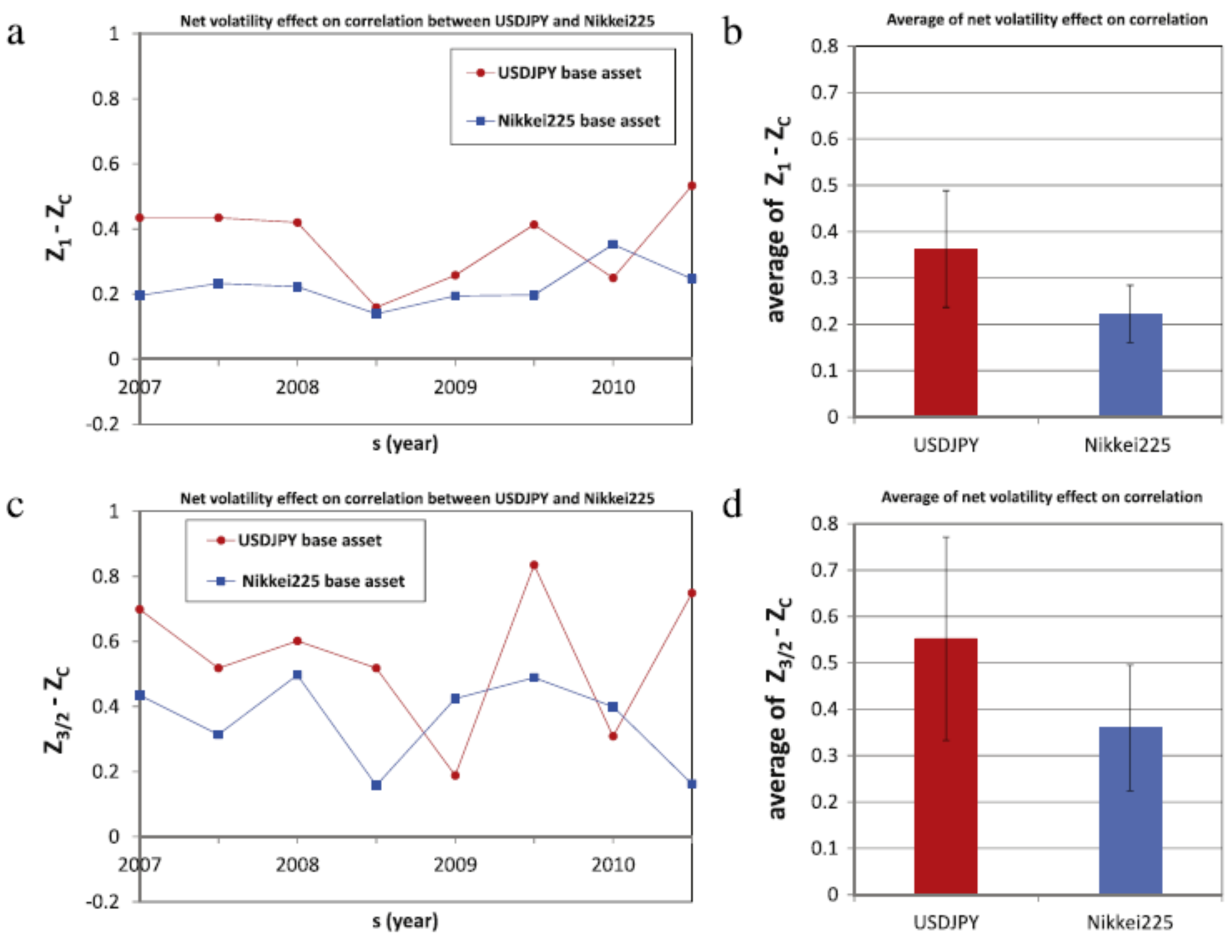
Constrained by Nikkei SD

1 sigma

1.5 sigma



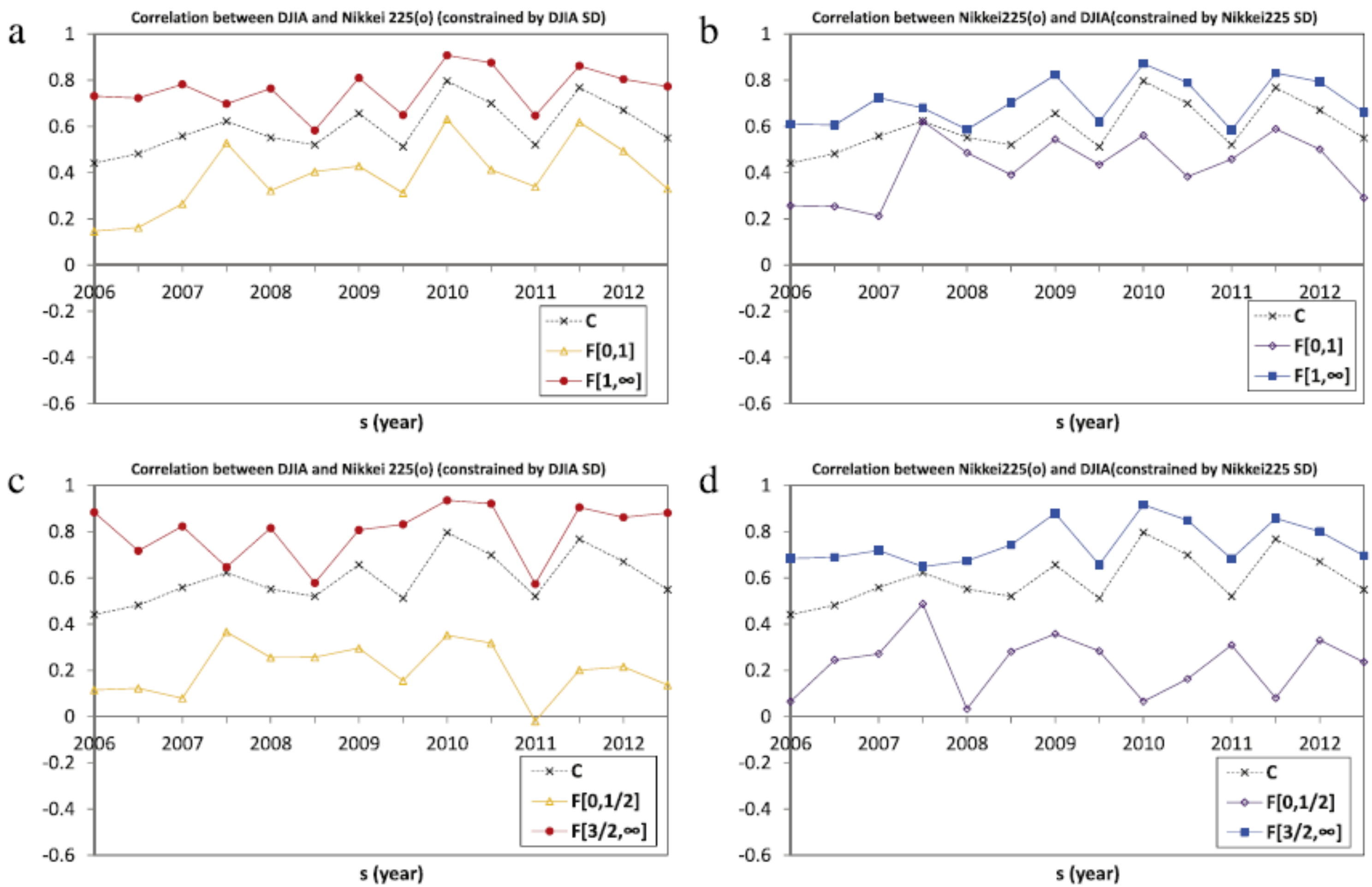
Directionality (USDJPY and Nikkei(o))



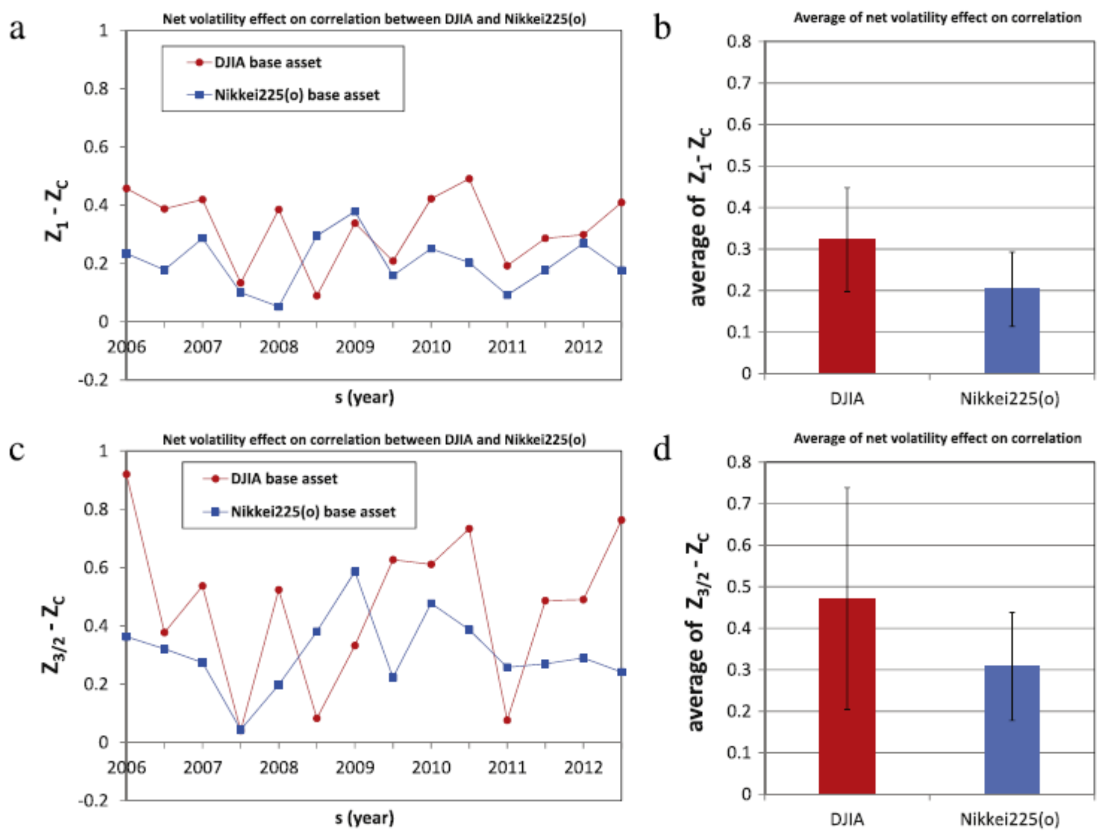
USDJPY > Nikkei

P value: 0.018 (top)
0.057 (bottom)

Empirical result: DJIA and Nikkei(o)



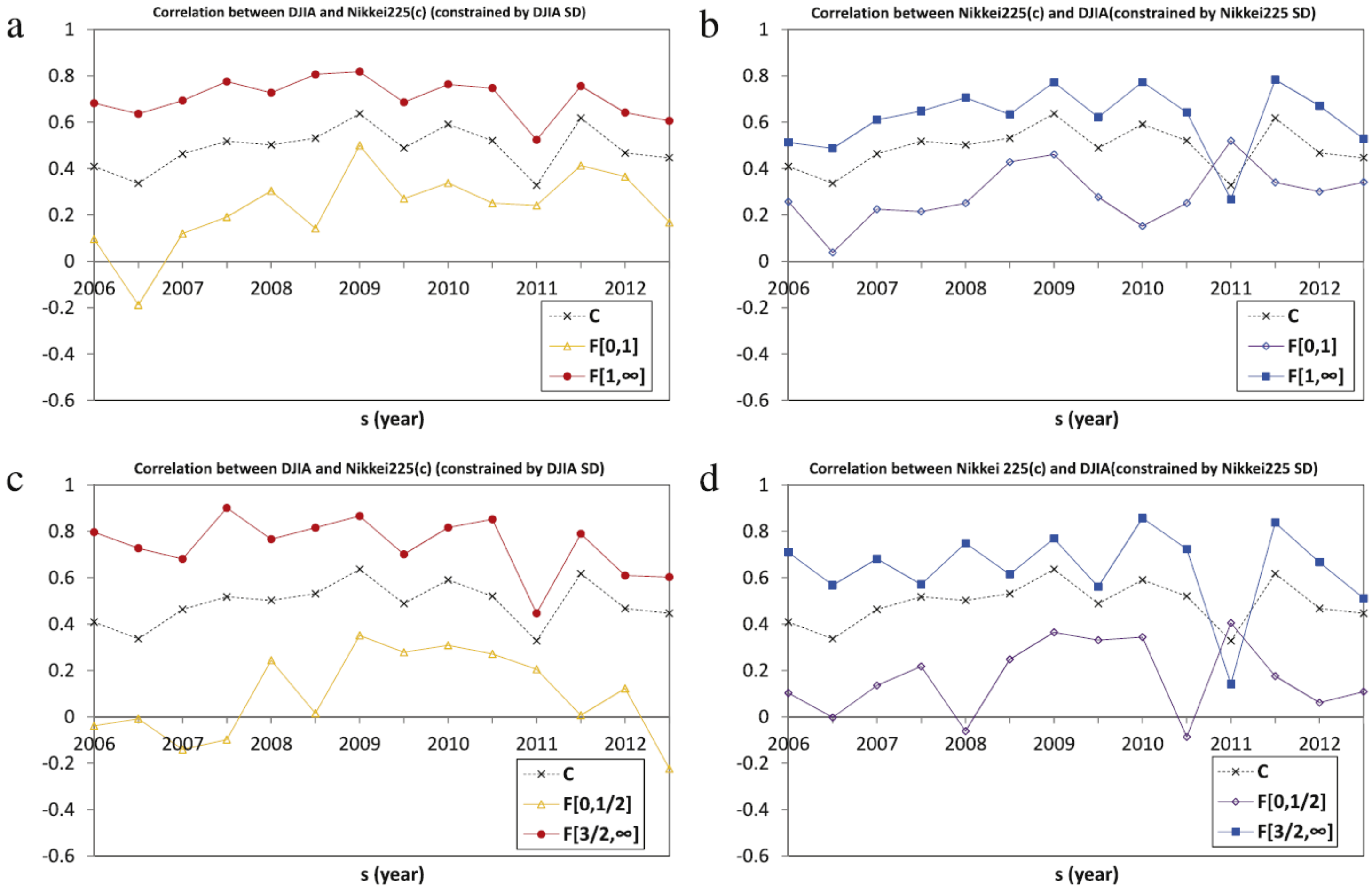
Directionality (DJIA and Nikkei(o))



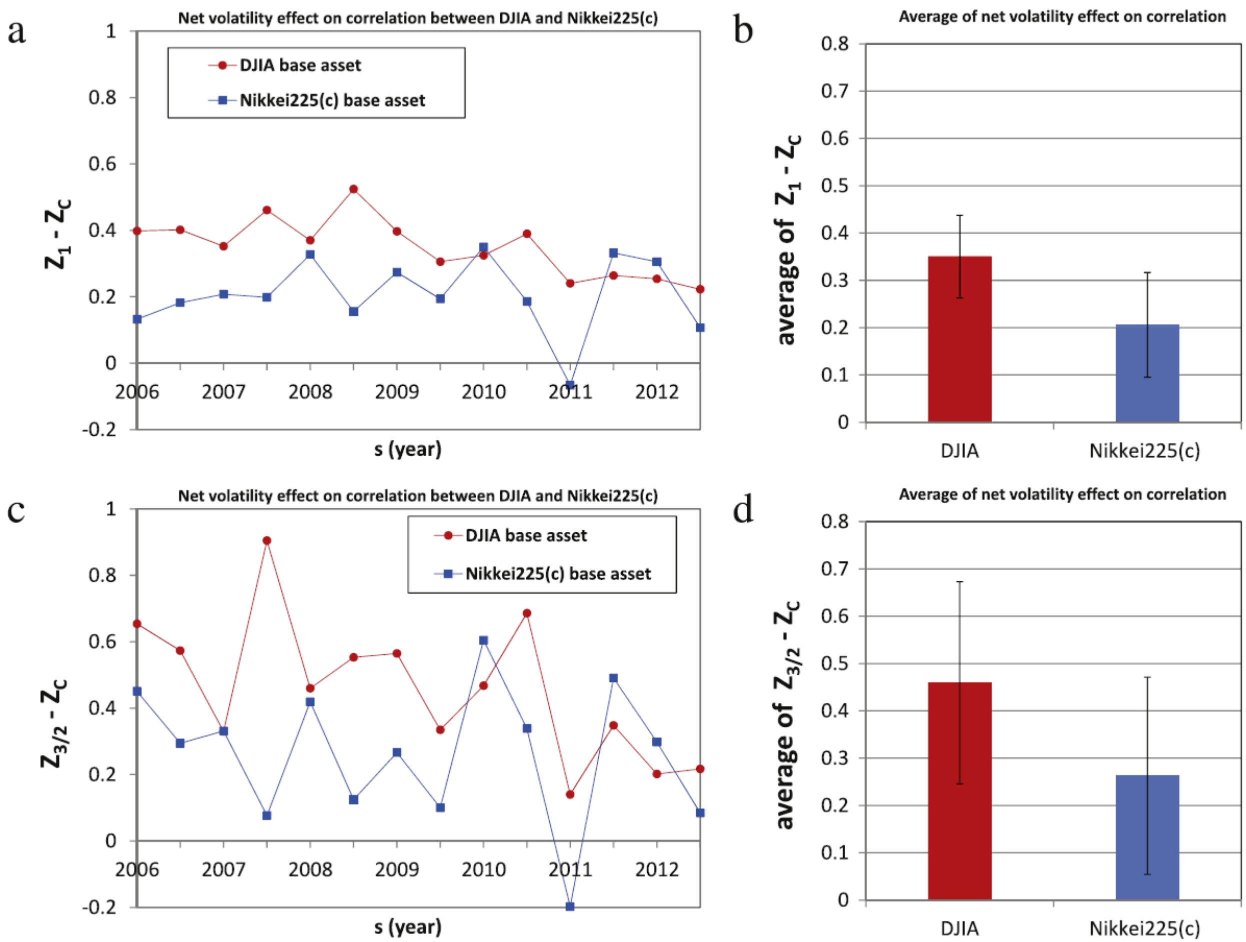
DJIA > Nikkei (o)

P value: 0.0079(top)
0.054 (bottom)

Empirical result: DJIA and Nikkei(c)



Directionality (DJIA and Nikkei(c))



DJIA > Nikkei (c)

P value: 0.00077(top)
0.021 (bottom)

Multivariate ARCH like model

- Let (R1(t), R2(t)) be two dimensional random variables which have a covariance matrix Dij(t) and zero mean vector.

$$D_{11}(t) = w_1 + \alpha_1 (R_1(t-1))^2$$

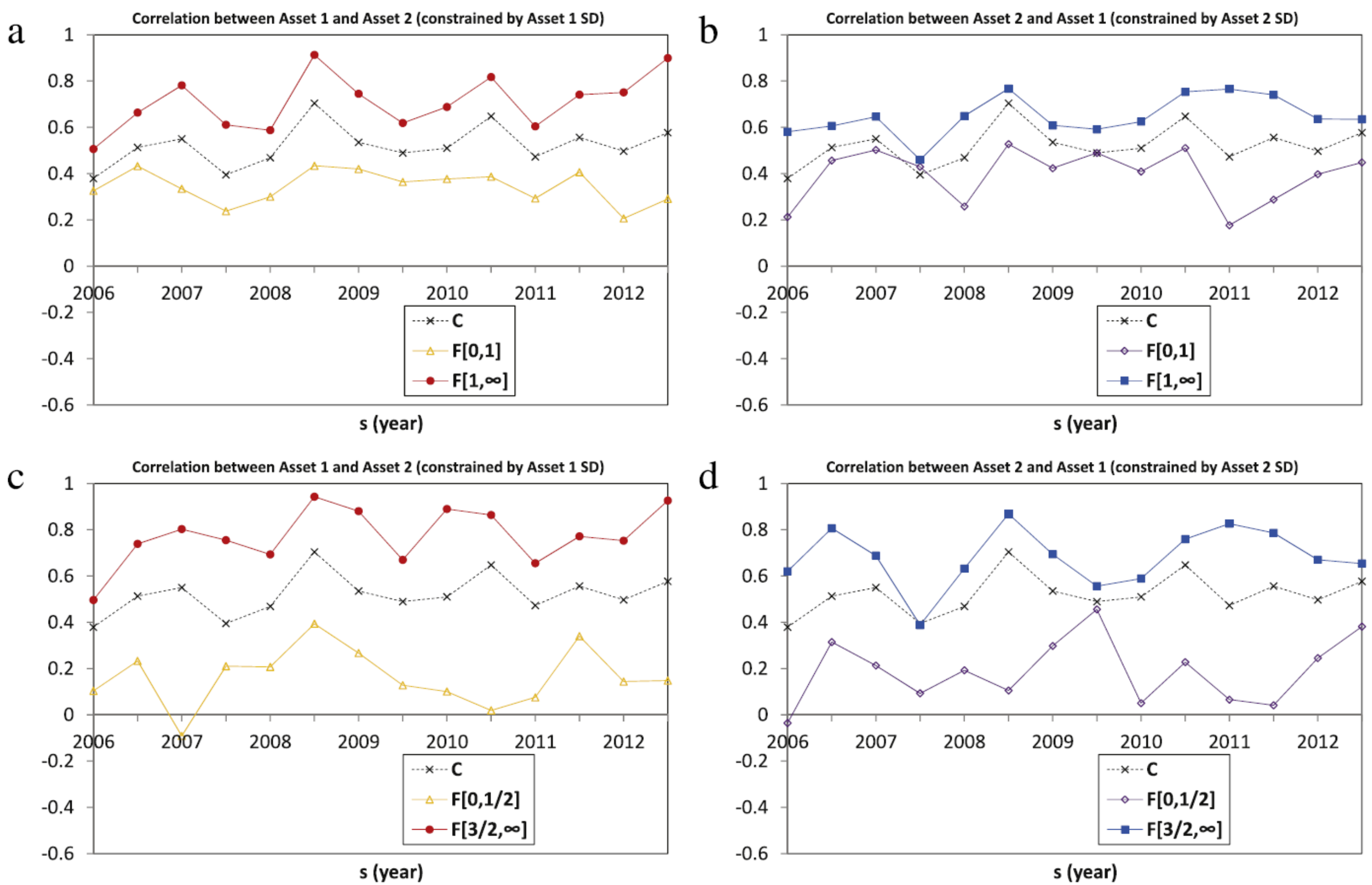
$$D_{22}(t) = w_2 + \alpha_2 (R_2(t-1))^2$$

$$D_{12}(t) = w_{12} + \beta_1 (R_1(t-1))^2 + \cancel{\beta_2 (R_2(t-1))^2}$$

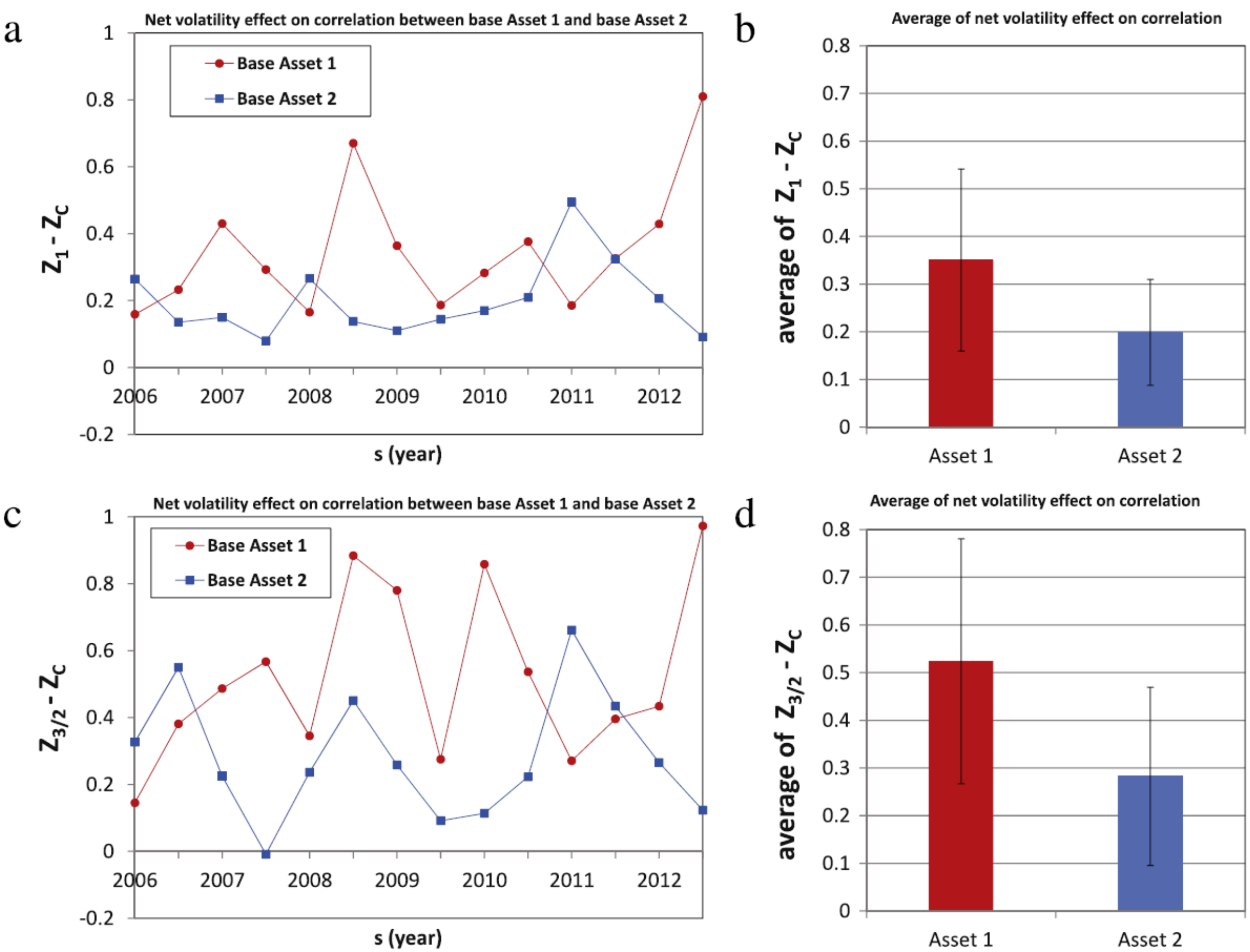
delete

By definition, the direction of influence is from Asset 1 to Asset 2

Simulated result



Simulated result(2)



Asset 1 > Asset 2

P value: 0.018(top)
0.0090(bottom)

Conclusion

- We have introduced the volatility-constrained correlation metric and examined the asymmetric feature of this metric with respect to the base asset.

- Unlike the standard correlation coefficient, our metric allows us to identify which asset is more influential than the other.

- In future research, it could be important to study the correlation among stocks, bonds, and currency markets with respect to this asymmetric feature and analyze the direction of the correlation between them.